

# Study $a_0^0(980)$ - $f_0(980)$ mixing from $a_0^0(980) \rightarrow f_0(980)$ transition

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## Abstract

Various processes have been proposed previously to study  $a_0^0(980)$ - $f_0(980)$  mixing through  $f_0(980) \rightarrow a_0^0(980)$  transition. Here we investigate in detail the difference between  $a_0^0(980) \rightarrow f_0(980)$  and  $f_0(980) \rightarrow a_0^0(980)$  transitions. It is found that the  $a_0^0(980) \rightarrow f_0(980)$  transition can provide additional constrains to the parameters of  $a_0^0(980)$  and  $f_0(980)$  mesons. Proposal is made to study  $a_0^0(980)$ - $f_0(980)$  mixing from  $\chi_{c1} \rightarrow \pi^0 a_0^0(980) \rightarrow \pi^0 f_0(980)$  reaction at the upgraded Beijing electron positron collider with the BESIII detector.

## I. INTRODUCTION

More than thirty years after their discovery, today the nature of light scalar mesons  $f_0(980)$  and  $a_0^0(980)$  is still in controversy. They have been described as quark-antiquark, four quarks,  $K\bar{K}$  molecule, quark-antiquark-gluon hybrid, and so on. Now the study of their nature has become a central problem in the light hadron spectroscopy.

In the late 1970s, the mixing between the  $a_0^0(980)$  and  $f_0(980)$  resonances was first suggested theoretically in Ref.[1]. Its mixing intensity is expected to shed important light on the nature of these two resonances, and has hence been studied extensively on its different aspects and possible manifestations in various reactions [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. But unfortunately no firm experimental determination on this quantity is available yet. Obviously, more solid and precise measurements on this quantity are needed, such as by polarized target experiment on the reaction  $\pi^- p \rightarrow \eta\pi^0 n$  [4],  $J/\psi$  decays [6, 15], and  $dd \rightarrow \alpha\eta\pi^0$  reactions from WASA at COSY [12]. In Ref. [15], we pointed out that the  $a_0^0(980)$ - $f_0(980)$  mixing intensity can be precisely measured through  $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0^0(980) \rightarrow \phi\eta\pi^0$  reaction at the upgraded Beijing electron positron collider with the BESIII detector.

In all these previous proposals, the  $f_0(980)$  is produced first, then transits to the  $a_0^0(980)$  by the  $a_0^0(980)$ - $f_0(980)$  mixing, i.e.,  $f_0(980) \rightarrow a_0^0(980)$  transition. In this article, we investigate in detail the difference between  $a_0^0(980) \rightarrow f_0(980)$  and  $f_0(980) \rightarrow a_0^0(980)$  transitions. We define two kinds of mixing intensities  $\xi_{fa}$  and  $\xi_{af}$  for the  $f_0(980) \rightarrow a_0^0(980)$  and  $a_0^0(980) \rightarrow f_0(980)$  transitions, respectively. We find there are some differences between them. We find that  $\xi_{af}$  has more dependence on the parameters of  $f_0(980)$ , especially the  $g_{fKK}/g_{f\pi\pi}$ , while the  $\xi_{fa}$  has more dependence on the parameters of  $a_0^0(980)$ , especially  $g_{aKK}/g_{a\pi\eta}$ . For this reason, using the reaction of  $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0^0(980) \rightarrow \phi\eta\pi^0$  to study only the  $f_0(980) \rightarrow a_0^0(980)$  mixing is not perfect enough. For better determination of all relevant parameters for the  $f_0(980)$  and  $a_0^0(980)$  mesons, it would be useful to find some reaction to study  $a_0^0(980) \rightarrow f_0(980)$  mixing in addition. Recently, CLEO Collaboration reported an experimental study of the  $\chi_{c1} \rightarrow \pi^+\pi^-\eta$  reaction [30]. The  $a_0^\pm(980)$  resonances are clearly showing up and dominant. From isospin symmetry, the  $\chi_{c1} \rightarrow \pi^0 a_0^0(980)$  should be produced with the same rate as  $\chi_{c1} \rightarrow \pi^\pm a_0^\mp(980)$ . This may provide a nice place for studying the  $a_0^0(980)$ - $f_0(980)$  mixing from  $a_0^0(980) \rightarrow f_0(980)$  transition by  $\chi_{c1} \rightarrow \pi^0 a_0^0(980) \rightarrow \pi^0 f_0(980) \rightarrow \pi^0\pi\pi$  reaction. From our estimation, more than

300 events can be reconstructed by the BESIII detector in the narrow peak with a width of about 8 MeV around the mass of 990 MeV in the  $\pi\pi$  invariant mass spectrum.

In the next section, we give a brief review of the theory for the  $a_0^0(980)$ - $f_0(980)$  mixing term. Then in the Sect.III we define two mixing intensities and tell the differences of mixing intensity. In Sect.IV we estimate the rate for the reaction of  $\chi_{c1} \rightarrow \pi^0\pi\pi$ . Finally we give a summary in Sect.V.

## II. THE $a_0^0(980)$ - $f_0(980)$ MIXING AMPLITUDE

The basic theory for the  $a_0^0(980)$ - $f_0(980)$  mixing was already pointed out by Achasov and collaborators [1]. For the nearly degenerate  $a_0^0(980)$  (isospin 1) and  $f_0(980)$  (isospin 0), both can decay into  $K\bar{K}$ . Due to isospin breaking effect, the charged and neutral kaon thresholds are different by about 8 MeV. Between the charged and neutral kaon thresholds the leading term to the  $a_0^0(980)$ - $f_0(980)$  mixing amplitude is dominated by the unitary cuts of the intermediate two-kaon system and proportional to the difference of phase spaces for the charged and neutral kaon systems.

Considering the  $a_0^0(980)$ - $f_0(980)$  mixing, the propagator of  $a_0^0(980)/f_0(980)$  can be expressed as [2] :

$$G = \frac{1}{D_f D_a - |D_{af}|^2} \begin{pmatrix} D_a & D_{af} \\ D_{af} & D_f \end{pmatrix}, \quad (1)$$

where  $D_a$  and  $D_f$  are the denominators for the usual propagators of  $a_0^0(980)$  and  $f_0(980)$ , respectively :

$$D_a = m_a^2 - s - i\sqrt{s}[\Gamma_{\eta\pi}^a(s) + \Gamma_{K\bar{K}}^a(s)], \quad (2)$$

$$D_f = m_f^2 - s - i\sqrt{s}[\Gamma_{\pi\pi}^f(s) + \Gamma_{K\bar{K}}^f(s)], \quad (3)$$

$$\Gamma_{bc}^a(s) = \frac{g_{abc}^2}{16\pi\sqrt{s}}\rho_{bc}(s), \quad (4)$$

$$\rho_{bc}(s) = \sqrt{[1 - (m_b - m_c)^2/s][1 - (m_b + m_c)^2/s]}. \quad (5)$$

The  $D_{af}$  is the mixing term. From [1, 4], the mixing due to  $K\bar{K}$  loops gives

$$D_{af,K\bar{K}} = \frac{g_{a_0^0(980)K^+K^-} g_{f_0(980)K^+K^-}}{16\pi} \left\{ i[\rho_{K^+K^-}(s) - \rho_{K^0\bar{K}^0}(s)] - \mathcal{O}(\rho_{K^+K^-}^2(s) - \rho_{K^0\bar{K}^0}^2(s)) \right\}. \quad (6)$$

Since the mixing is mainly coming from the  $K\bar{K}$  loops, we have  $D_{af} \approx D_{af,K\bar{K}}$ , and this is the amplitude of  $a_0^0(980)$ - $f_0(980)$  mixing. From Eq.(6), the  $D_{af}$  becomes large only when the  $\sqrt{s}$  is between the  $2M_{K^+}$  and  $2M_{K^0}$ , so it is a narrow peak with the width of about  $8MeV$ .

### III. TWO TYPES OF REACTION AND MIXING INTENSITY

#### A. Two types of reaction of $a_0^0(980)$ - $f_0(980)$ mixing

There are two types of reaction which can be used to study  $a_0^0(980)$ - $f_0(980)$  mixing :  $X \rightarrow Y f_0(980) \rightarrow Y a_0^0(980) \rightarrow Y \pi^0 \eta$  and  $X \rightarrow Y a_0^0(980) \rightarrow Y f_0(980) \rightarrow Y \pi \pi$ .

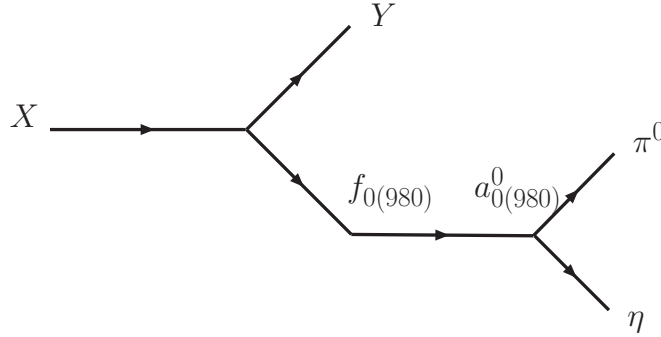


FIG. 1: The Feynman diagram of  $X \rightarrow Y f_0(980) \rightarrow Y a_0^0(980) \rightarrow Y \pi^0 \eta$ .

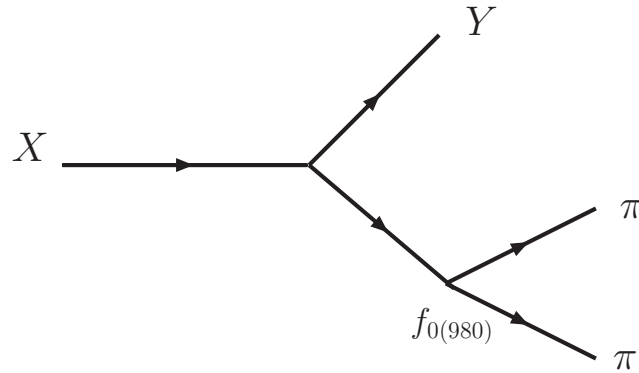


FIG. 2: The Feynman diagram of  $X \rightarrow Y f_0(980) \rightarrow Y \pi \pi$ .

For the reaction  $X \rightarrow Y f_0(980) \rightarrow Y a_0^0(980) \rightarrow Y \pi^0 \eta$  as shown by the Feynman diagram in Fig.1, the influence of various X and Y on the  $a_0^0(980)$ - $f_0(980)$  mixing can be removed by

its comparison to the corresponding reaction  $X \rightarrow Y f_0(980) \rightarrow Y \pi \pi$  as shown in Fig.2. We define the mixing intensity  $\xi_{fa}$  for the  $f_0(980) \rightarrow a_0^0(980)$  transition as the following

$$\xi_{fa}(s) = \frac{d\Gamma_{X \rightarrow Y f_0(980) \rightarrow Y a_0^0(980) \rightarrow Y \pi^0 \eta(s)}}{d\Gamma_{X \rightarrow Y f_0(980) \rightarrow Y \pi \pi(s)}}, \quad (7)$$

where  $s$  is the invariant mass squared of two mesons in final state. With Eqs.(1-6), one can get the  $\xi_{fa}(s)$  as:

$$\xi_{fa}(s) = \frac{|D_{af}|^2 \Gamma_{\pi\eta}^a}{|D_a|^2 \Gamma_{\pi\pi}^f} \quad (8)$$

$$= \frac{\left| \frac{g_{a_0^0 K^+ K^-} - g_{f_0 K^+ K^-}}{g_{a_0^0 \pi^0 \eta} g_{f_0 \pi^0 \pi^0}} \right|^2 \frac{|\rho_{K^+ K^-}(s) - \rho_{K^0 \bar{K}^0}(s)|^2}{3\rho_{\pi\pi}(s)\rho_{\pi\eta}(s)}}{1} \times \frac{1}{\left| \frac{m_a^2 - s}{\Gamma_{\pi\eta}^a \sqrt{s}} - i \left[ \left| \frac{g_{a_0^0 K^+ K^-}}{g_{a_0^0 \pi^0 \eta}} \right|^2 \left( \frac{\rho_{K^+ K^-}(s)}{\rho_{\pi\eta}(s)} + \frac{\rho_{K^0 \bar{K}^0}(s)}{\rho_{\pi\eta}(s)} \right) + 1 \right] \right|^2}. \quad (9)$$

Similarly, for the reaction  $X \rightarrow Y a_0^0(980) \rightarrow Y f_0(980) \rightarrow Y \pi \pi$ , we define the mixing intensity  $\xi_{af}$  for the  $a_0^0(980) \rightarrow f_0(980)$  transition and get its formula as the following

$$\xi_{af}(s) = \frac{d\Gamma_{X \rightarrow Y a_0^0(980) \rightarrow Y f_0(980) \rightarrow Y \pi \pi(s)}}{d\Gamma_{X \rightarrow Y a_0^0(980) \rightarrow Y \pi^0 \eta(s)}}, \quad (10)$$

$$= \frac{\left| \frac{g_{a_0^0 K^+ K^-} - g_{f_0 K^+ K^-}}{g_{a_0^0 \pi^0 \eta} g_{f_0 \pi^0 \pi^0}} \right|^2 \frac{|\rho_{K^+ K^-}(s) - \rho_{K^0 \bar{K}^0}(s)|^2}{3\rho_{\pi\pi}(s)\rho_{\pi\eta}(s)}}{1} \times \frac{1}{\left| \frac{m_f^2 - s}{\Gamma_{\pi\pi}^f \sqrt{s}} - i \left[ \left| \frac{g_{f_0 K^+ K^-}}{g_{f_0 \pi^0 \pi^0}} \right|^2 \left( \frac{\rho_{K^+ K^-}(s)}{3\rho_{\pi\pi}(s)} + \frac{\rho_{K^0 \bar{K}^0}(s)}{3\rho_{\pi\pi}(s)} \right) + 1 \right] \right|^2}. \quad (11)$$

We can redefine that:

$$R_f = \left| g_{f_0(980) K^+ K^-} / g_{f_0(980) \pi^0 \pi^0} \right|^2, \quad R_a = \left| g_{a_0^0(980) K^+ K^-} / g_{a_0^0(980) \pi^0 \eta} \right|^2, \quad (12)$$

$$A_{(s)}^f = \rho_{K^+ K^-}(s) / 3\rho_{\pi\pi}(s), \quad A_{(s)}^a = \rho_{K^+ K^-}(s) / \rho_{\pi\eta}(s), \quad (13)$$

$$B_{(s)}^f = \rho_{K^0 \bar{K}^0}(s) / 3\rho_{\pi\pi}(s), \quad B_{(s)}^a = \rho_{K^0 \bar{K}^0}(s) / \rho_{\pi\eta}(s), \quad (14)$$

$$C_{(s)}^f = (m_f^2 - s) / \Gamma_{\pi\pi}^f \sqrt{s}, \quad C_{(s)}^a = (m_a^2 - s) / \Gamma_{\pi\eta}^a \sqrt{s}, \quad (15)$$

$$H_{(s)} = |\rho_{K^+ K^-}(s) - \rho_{K^0 \bar{K}^0}(s)|^2 / (3\rho_{\pi\pi}(s)\rho_{\pi\eta}(s)). \quad (16)$$

The  $D_{af}$  becomes large when the  $\sqrt{s}$  is between the  $2M_{K^+}$  and  $2M_{K^0}$ . In this mass range,

$A_{(s)}^f$  and  $A_{(s)}^a$  are real, meanwhile  $B_{(s)}^f$  and  $B_{(s)}^a$  are imaginary; then  $\xi_{af}$  and  $\xi_{fa}$  become:

$$\xi_{af}(s) = \frac{R_a R_f \times H_{(s)}}{\left(C_{(s)}^f + R_f \times |B_{(s)}^f|\right)^2 + \left(1 + R_f \times A_{(s)}^f\right)^2}, \quad (17)$$

$$\xi_{fa}(s) = \frac{R_a R_f \times H_{(s)}}{\left(C_{(s)}^a + R_a \times |B_{(s)}^a|\right)^2 + \left(1 + R_a \times A_{(s)}^a\right)^2}. \quad (18)$$

### B. Predictions of $\xi_{af}$ and $\xi_{fa}$ from various models and experiment information

From equations given above, one can see that the mixing intensity  $|\xi|$  depends on  $g_{a_0^0(980)K^+K^-}$ ,  $g_{f_0(980)K^+K^-}$ ,  $g_{a_0^0(980)\pi^0\eta}$ ,  $g_{f_0(980)\pi^0\pi^0}$ ,  $m_f$  and  $m_a$ . Various models for the structures of  $a_0^0(980)$  and  $f_0(980)$  give different predictions for these coupling constants and mass [14, 17, 18, 20] as listed in Table I by No.A-D. There have also been some experimental measurements on these coupling and mass constants [21, 22, 23, 24, 25, 26, 27, 28] as listed by No.E-H. The corresponding predictions for the  $\xi_{af}$  and  $\xi_{fa}$  from these various theoretical and experimental values of the coupling constants are calculated. In the calculation, the masses for  $K^+$ ,  $K^0$ ,  $\pi^0$  and  $\eta$  are taken from PDG2008 [29] as  $m_{K^+} = 493.7$  MeV,  $m_{K^0} = 497.7$  MeV,  $m_\pi = 135.0$  MeV and  $m_\eta = 547.5$  MeV, respectively. We give the value of  $\xi_{af}$  and  $\xi_{fa}$  at the  $\sqrt{s} = 991.4$  MeV in Table I and the dependence of  $a_0^0(980)$ - $f_0(980)$  mixing intensities  $\xi_{af}$  and  $\xi_{fa}$  vs two-meson invariant mass in Fig.(3). There is obviously some difference between these two mixing intensities.

### C. Discussion on the difference of two mixing intensities $\xi_{af}$ and $\xi_{fa}$

We know from Eqs.(17,18), if the  $C_{(s)}^i$ ,  $R_i \times B_{(s)}^i$  and  $R_i \times A_{(s)}^i$  (i=a or f) are much smaller than 1, we will have  $\xi_{af}(s) \simeq \xi_{fa}(s) \simeq R_a R_f \times H_{(s)}$ . Then from these mixing intensities we only get the  $R_a R_f$  no matter which type of reactions one makes the measurement.

However, are these quantities really much smaller than 1 ? For  $\sqrt{s}$  between the  $2M_{K^+}$  and  $2M_{K^0}$ ,  $\sqrt{s} \approx m_i$ , we have

$$C_{(s)}^i = (m_i^2 - s)/(\Gamma_{\pi\pi(\eta)}^i \sqrt{s}) = (m_i - \sqrt{s})(m_i + \sqrt{s})/(\Gamma_{\pi\pi(\eta)}^i \sqrt{s}) \approx 2(m_i - \sqrt{s})/\Gamma_{\pi\pi(\eta)}^i. \quad (19)$$

Here  $\Gamma_{\pi\pi(\eta)}^i$  is about 100 MeV from many experiments,  $m_i \simeq 980$  MeV, then one gets  $C_{(4M_{K^+}^2)}^i = -0.14$  and  $C_{(4M_{K^0}^2)}^i = -0.30$ , which are not so small.

No.	model or experiment	$m_a$	$g_{a_0\pi\eta}$	$g_{a_0K^+K^-}$	$m_f$	$g_{f_0\pi^0\pi^0}$	$g_{f_0K^+K^-}$	$ \xi_{fa} $	$ \xi_{af} $
A	$q\bar{q}$ model [14]	983	2.03	1.27	975	0.64	1.80	0.023	0.010
B	$q^2\bar{q}^2$ model [14]	983	4.57	5.37	975	1.90	5.37	0.068	0.062
C	$K\bar{K}$ model [17, 18, 19]	980	1.74	2.74	980	0.65	2.74	0.21	0.15
D	$q\bar{q}g$ model [20]	980	2.52	1.97	975	1.54	1.70	0.005	0.006
E	SND [21, 22]	995	3.11	4.20	969.8	1.84	5.57	0.088	0.089
F	KLOE [23, 24]	984.8	3.02	2.24	973	2.09	5.92	0.034	0.025
G	BNL [25]	1001	2.47	1.67	953.5[26]	1.36[26]	3.26[26]	0.019	0.014
H	CB [27]	999	3.33	2.54	965[28]	1.66[28]	4.18 [28]	0.027	0.023

TABLE I:  $m_{a_0^0(980)}$ (MeV),  $m_{f_0(980)}$ (MeV) and coupling constants  $g_{a_0\pi\eta}$ (GeV),  $g_{a_0K^+K^-}$ (GeV),  $g_{f_0K^+K^-}$ (GeV) and  $g_{f_0\pi^0\pi^0}$ (GeV) from various models (A-D) and experimental measurements (E-H), and calculated values of  $|\xi_{af}|$  and  $|\xi_{fa}|$  at the  $\sqrt{s} = 991.4$  MeV by Eqs.(17,18).

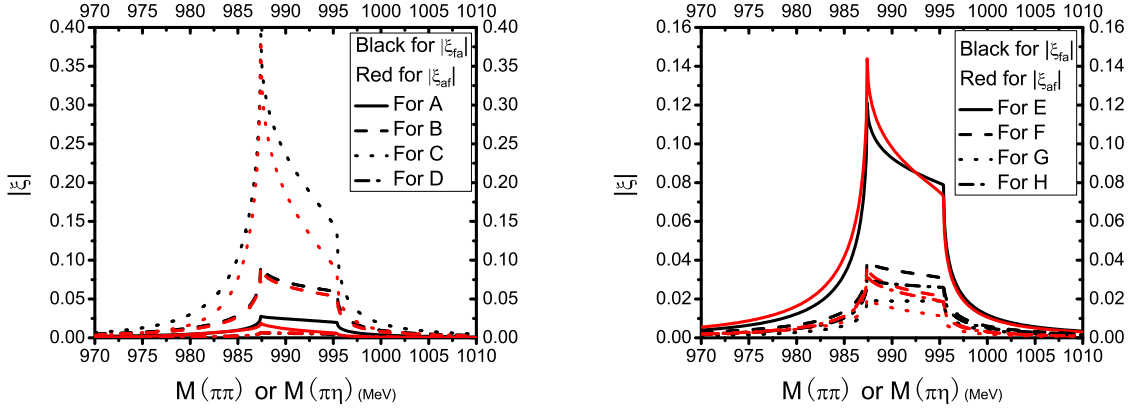


FIG. 3: Predictions for the  $a_0^0(980)$ - $f_0(980)$  mixing intensity  $\xi_{af}$  and  $\xi_{fa}$  vs two-meson invariant mass from various models A-D (left) and various experimental measured parameters E-H (right).

For  $R_i \times B_{(s)}^i$  and  $R_i \times A_{(s)}^i$  with  $A_{(4M_{K^+}^2)}^i = |B_{(4M_{K^0}^2)}^i| = 0$ ,  $A_{(4M_{K^0}^2)}^a \simeq |B_{(4M_{K^+}^2)}^a| = 0.201$  and  $A_{(4M_{K^0}^2)}^f \simeq |B_{(4M_{K^+}^2)}^f| = 0.044$ , since from Table I the smallest  $R_a$  and  $R_f$  are 0.5 and 5.7 respectively among various experimental determinations, the  $R_i \times |B_{(s)}^i|$  and  $R_i \times A_{(s)}^i$  are also larger than 0.1 and are not small enough to be neglected.

Then from Eqs.(17,18), one can see that besides the common numerator, the  $\xi_{af}$  has

additional dependence on the parameters of  $f_0(980)$  while the  $\xi_{fa}$  has additional dependence on the parameters of  $a_0^0(980)$ .

From above analysis, we understand why  $\xi_{af}$  is different from  $\xi_{fa}$  as shown in Table I and Fig.3. Both mixing intensities depend on six parameters:  $m_f$ ,  $m_a$ ,  $g_{a_0^0(980)K^+K^-}$ ,  $g_{f_0(980)K^+K^-}$ ,  $g_{a_0^0(980)\pi^0\eta}$ ,  $g_{f_0(980)\pi^0\pi^0}$ , which are all important for understanding the nature of the  $a_0^0(980)$  and  $f_0(980)$  mesons, but are not well determined yet. Therefore to measure  $\xi_{af}$  in addition to  $\xi_{fa}$  will be very useful for pinning down these parameters.

To further demonstrate the importance of measuring the  $\xi_{af}$  in addition to the  $\xi_{fa}$ , a typical example is given as the following.

No.	$m_a$ (MeV)	$g_{a_0\pi\eta}$ (GeV)	$g_{a_0K^+K^-}$ (GeV)	$m_f$ (MeV)	$g_{f_0\pi^0\pi^0}$ (GeV)	$g_{f_0K^+K^-}$ (GeV)
1	980	3.2	4.2	980	1.5	4.0
2	980	3.2	3.0	980	1.5	5.12

TABLE II: Two typical parameter sets for  $a_0^0(980)$  and  $f_0(980)$ .

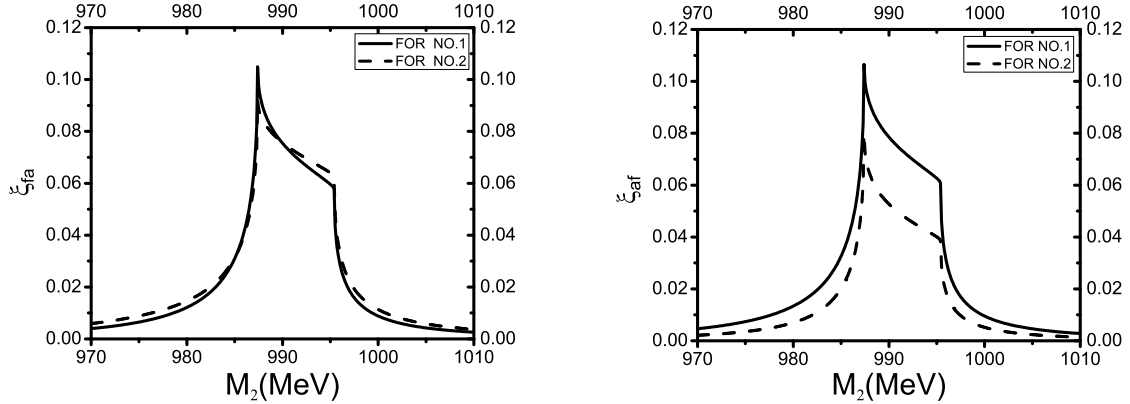


FIG. 4: Diagrams of  $a_0^0(980)$ - $f_0(980)$  mixing intensity  $\xi_{af}$  and  $\xi_{fa}$  vs two-meson invariant mass with parameter sets No.1 and No.2 of Table II.

For two sets of parameters given in Table.II, we can see that the set No.1 is close to the SND values in Table.I. The set No.2 changes the not well measured  $g_{aKK}$  and  $g_{fKK}$  in their experimental uncertainties. We plot the corresponding diagrams of  $a_0^0(980)$ - $f_0(980)$  mixing intensities  $\xi_{af}$  and  $\xi_{fa}$  vs two-meson invariant mass  $M_2$  as shown in the Fig.4. The two sets of parameters give almost identical  $\xi_{fa}$  but very different  $\xi_{af}$ .



#### IV. POSSIBILITY OF MEASURING $\xi_{af}$ FROM $\chi_{c1} \rightarrow \pi^0 a_0^0(980) \rightarrow \pi^0 f_0(980) \rightarrow \pi^0 \pi \pi$

The Feynman diagram for the reaction  $\chi_{c1} \rightarrow \pi^0 a_0^0(980) \rightarrow \pi^0 f_0(980) \rightarrow \pi^0 \pi \pi$  is shown in Fig 5.

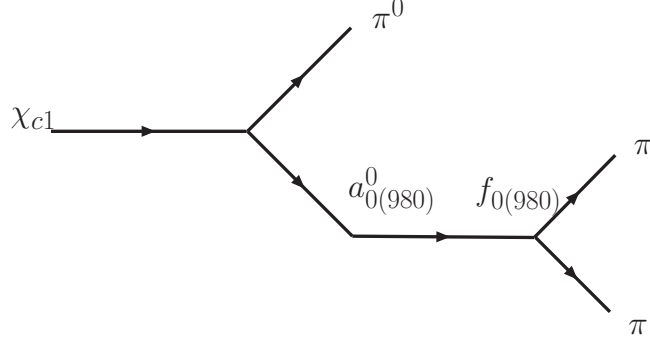


FIG. 5: Feynman diagram for  $\chi_{c1} \rightarrow \pi^0 a_0^0(980) \rightarrow \pi^0 f_0(980) \rightarrow \pi^0 \pi \pi$

The invariant amplitude for this reaction is

$$\mathcal{M}_{\chi_{c1} \rightarrow \pi^+ \pi^- \pi^0} = g_{\chi_{c1} a_0^0(980) \pi^0} \varepsilon_{\chi_{c1}}^\mu (p_{\pi^0} - p_{f_0(980)}) \frac{D_{af}}{D_f D_a} \sqrt{3} g_{f_0(980) \pi^0 \pi^0} \quad (20)$$

The coupling constant  $g_{\chi_{c1} a_0^0(980) \pi^0}$  can be determined by the reaction  $\chi_{c1} \rightarrow a_{0(980)}^\pm \pi^\mp$ .

$$|g_{\chi_{c1} a_0^0(980) \pi^0}|^2 = |g_{\chi_{c1} a_{0(980)}^+ \pi^-}|^2 = \frac{12\pi M_{\chi_{c1}}^2 \Gamma_{\chi_{c1}} Br_{(\chi_{c1} \rightarrow a_{0(980)}^+ \pi^-)}}{|\vec{p}_{a_{0(980)}^+}|^3}. \quad (21)$$

According to Ref. [29],  $M_{\chi_{c1}} = 3510.66$  MeV,  $\Gamma_{\chi_{c1}} = 0.89$  MeV, and  $Br_{\chi_{c1} \rightarrow \eta \pi^+ \pi^-} = (5.2 \pm 0.6) \times 10^{-3}$ . The  $\chi_{c1} \rightarrow a_{0(980)}^\pm \pi^\mp$  gives the dominant contribution of about 75.1% [30]. So we have  $Br_{(\chi_{c1} \rightarrow a_{0(980)}^+ \pi^-)} \simeq 0.0052 \times 0.751/2 \simeq 2 \times 10^{-3}$ . From the formulae of Eqs.(1-6) and parameters of setting No.H listed in Table 1, we can calculate the  $Br_{\chi_{c1} \rightarrow \pi^0 a_0^0(980) \rightarrow \pi^0 f_0(980) \rightarrow \pi^0 \pi \pi}$ . It is  $4.6 \times 10^{-6}$  and the invariant mass spectrum of  $\pi^+ \pi^-$  for  $\chi_{c1} \rightarrow \pi^0 a_0^0(980) \rightarrow \pi^0 f_0(980) \rightarrow \pi^0 \pi^+ \pi^-$  is shown in Fig.6.

The branching ratio of reaction  $\psi_{2s} \rightarrow \gamma \chi_{c1}$  is 0.088 [29]. At the upgraded Beijing electron positron collider with BESIII detector, about  $3.2 \times 10^9$   $\psi_{2s}$  events and hence about  $2.8 \times 10^8$   $\chi_{c1}$  events can be collected per year. From the branching ratio of  $\chi_{c1} \rightarrow \pi^0 \pi \pi = 4.6 \times 10^{-6}$ , more than 1200 events are expected to be collected. Considering the reconstruction efficiency of 30%, more than 300 events can be reconstructed for this channel. Since all these events should concentrate in a narrow region of about 8 MeV around 990 MeV in the  $\pi^+ \pi^-$  invariant

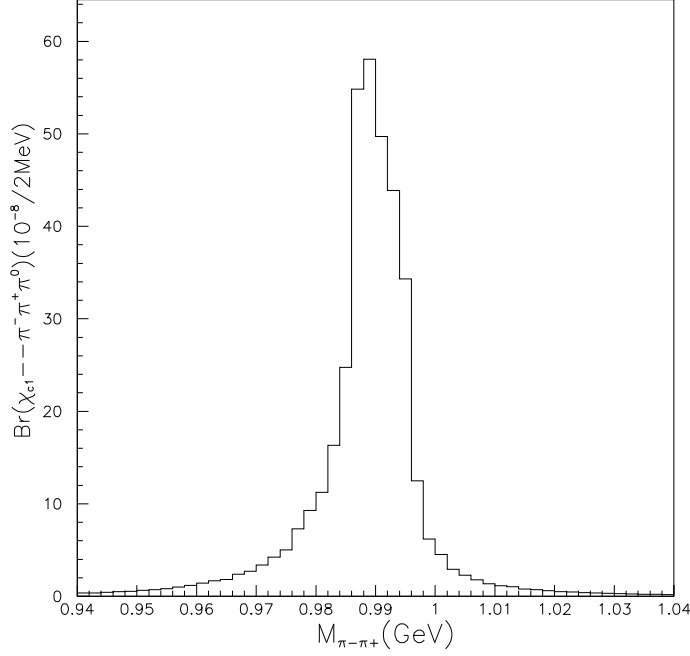


FIG. 6:  $\pi^+\pi^-$  invariant mass spectrum for  $\chi_{c1} \rightarrow \pi^0 a_0^0(980) \rightarrow \pi^0 f_0(980) \rightarrow \pi^0 \pi^+ \pi^-$

mass spectrum, the narrow peak should be easily observed, hence the  $\xi_{af}$  should be able to be measured by this reaction.

## V. SUMMARY

Various processes have been proposed previously to study  $a_0^0(980)$ - $f_0(980)$  mixing through  $f_0(980) \rightarrow a_0^0(980)$  transition. In this article we investigate in detail the difference between  $a_0^0(980) \rightarrow f_0(980)$  and  $f_0(980) \rightarrow a_0^0(980)$  transitions. Two corresponding mixing intensities  $\xi_{af}$  and  $\xi_{fa}$  are defined. It is found that besides the common numerator, the  $\xi_{af}$  has additional dependence on the parameters of  $f_0(980)$  while the  $\xi_{fa}$  has additional dependence on the parameters of  $a_0^0(980)$ . Therefore to measure  $a_0^0(980) \rightarrow f_0(980)$  transition in addition to the  $f_0(980) \rightarrow a_0^0(980)$  transition will be very useful for pinning down these parameters. We examine the possibility of measuring the  $a_0^0(980)$ - $f_0(980)$  mixing from  $\chi_{c1} \rightarrow \pi^0 \pi \pi$  for  $\xi_{af}$  at the upgraded Beijing Electron Positron Collider with BESIII detector and find it is feasible.

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